INF 240 - Exercise problems - 7

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Some of the following exercises are taken from Lidl & Niederreiter's *Finite fields*.

Exercise 1. *Let $\mathbb{F}$ be a finite field with more than 2 elements. Show that the sum of all elements of $\mathbb{F}$ is equal to 0.*

Hint: How does the sum of all elements in $\mathbb{F}$ change if we multiply it by a non-zero element?

Exercise 2. *Let a and b be elements of the finite field $\mathbb{F}_{2^n}$ for n odd. Show that $a^2 + ab + b^2 = 0$ implies $a = b = 0$.*

Hint: The polynomial $x^2 + x + 1$ is irreducible over $\mathbb{F}_2$, but it splits into linear factrors in $\mathbb{F}_{2^2}$.

Exercise 3. *Determine all primitive elements of the following finite fields:*

1. $\mathbb{F}_7$;

2. $\mathbb{F}_{17}$;

3. $\mathbb{F}_9$.

Exercise 4. *Let $\mathbb{F}$ be a field. Show that every finite subgroup of the multiplicative group $\mathbb{F}^*$ is cyclic.*

Exercise 5. *Let $\mathbb{F}$ be a field. Show that if the multiplicative group $\mathbb{F}^*$ is cyclic, then $\mathbb{F}$ is finite.*

Hint: Consider the multiplicative inverses of the elements in $\mathbb{F}^*$.

Exercise 6. *Find all subfields of $\mathbb{F}_{5^{42}}$.*